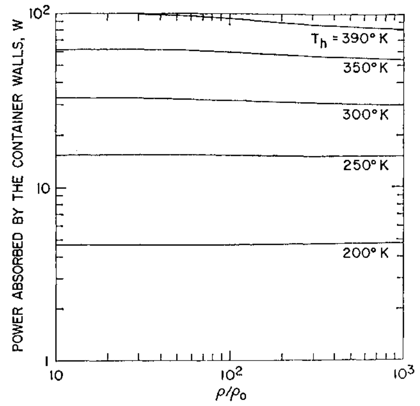

Fig. 4 Power absorbed by the container walls vs heater temperature and gas density.

no rotational structure because of the high densities involved. Hence their absorption coefficients are smoothly varying functions of wave number. This allows the absorption along any ray in the fluid to be evaluated from Eq. (2):

$$I_{\text{abs}} = \epsilon \int_{\nu_1}^{\nu_2} B_\nu(1 - e^{-\tau_\nu})d\nu \tag{2}$$

where B_ν = blackbody function; ν = wave number, cm^{-1}; and τ_ν = optical depth: $\tau_\nu = \alpha_\nu(\rho/\rho_0)^2L$, where α_ν = spectral absorption coefficient, amagat^{-2}-cm^{-1}, ρ/ρ_0 = gas density, amagat, and L = path length, cm.

The integration limits ν_1 and ν_2 correspond to wave numbers at which the spectral absorption coefficients are taken as zero. To calculate the absorption from the band tails (i.e., that portion of each molecular band that had absorption coefficients too small to be experimentally measured) exponential tails were added to the strongest absorption band. Calculations show that these tail bands never contributed more than 20% to the total absorbed power and hence could safely be neglected.

Figure 1 is a plot of the fractional absorption (f_{abs}) for a distance equal to R, the inside radius of the tank. This fraction is defined as I_{abs} normalized by the emission intensity of the heater in the pertinent spectral regin for both the 1556 and 3102 cm^{-1} absorption bands and is plotted vs ρ/ρ_0 for various T_h's. It may be seen that for ρ/ρ_0 between 10^2 and 10^3 amagat (i.e., for most operational conditions), the 1556 absorption band is strongly absorbed while the 3102 absorption band is only weakly absorbed.

Because of the moderate coefficient of absorption of the tank wall, much of the radiation in the 3102 cm^{-1} spectral range may be reflected or scattered back into the oxygen. This means that appreciably more of the radiated heater power in this spectral range may be ultimately absorbed by the oxygen. It can be shown that an upper limit to the power absorbed by the oxygen is given by

$$I_{\text{abs}}\ (3102\ \text{cm}^{-1}\ \text{band}) \leq \frac{\bar{\alpha}(\rho/\rho_0)^2R(2-\epsilon)}{1-(1-\epsilon)[1-2\bar{\alpha}(\rho/\rho_0)^2R]}\cdot \pi\epsilon A\int_{\nu_1}^{\nu_2} B_\nu d\nu \tag{3}$$

where $\bar{\alpha}$ is defined by

$$(1 - e^{-\bar{\alpha}(\rho/\rho_0)^2R})\int_{\nu_1}^{\nu_2} B_\nu d\nu = \int_{\nu_1}^{\nu_2} B_\nu(1 - e^{-\bar{\alpha}_\nu(\rho/\rho_0)R})d\nu$$

where $\epsilon = 0.32$. When Eq. (3) was evaluated for 200°K $\leq T_h \leq$ 500°K, and $10 \leq \rho/\rho_0 \leq 1000$, it was found that I_{abs} (3102 cm^{-1} band) was always less than 1% of I_{abs} (1556 cm^{-1} band). Hence, it too may be neglected.

Figure 2 shows the total infrared power absorbed from the heater. Even under the circumstances most favorable to absorption (i.e., at the very highest densities and heater temperatures) the upper limit to I_{abs} will be less than 20 w. The reasons that I_{abs} is so small are that both oxygen absorption bands are narrow and that the absorption coefficients of the 3102 cm^{-1} band are very low. This is shown in Fig. 3 where the spectral distribution of the blackbody function and the spectral extent of the absorption bands are plotted against wave number.

Since the oxygen absorbs only a small fraction of the heater radiation, most of the power radiated by the heater is absorbed by the container wall, which heats the oxygen by conduction. (The heat-transfer rate from the wall to the outside is very small and may be neglected.) As Kamat[4] has pointed out, supplying heat to the oxygen by conduction from the container wall is a very good way of minimizing temperature gradients and preventing pressure collapse in super critical oxygen storage systems. Hence, to maximize this method of heat transfer, the heater should be operated in such a way as to deliver the largest possible fraction of its dissipated power as radiation. Figure 4 shows the power absorbed by the walls as a function of ρ/ρ_0 and T_h for the heater after it has been blackened to produce an ϵ of 1.00. It can be seen that at $T_h \geq 350$°K, more than half the heat dissipated by the heater will be transferred to the oxygen by conduction from the container wall. Under such circumstances, the temperature gradients throughout the fluid will be minimized and the possibility of pressure collapse will be diminished.

In conclusion, it has been shown that in cryogenic oxygen storage systems employing a central heater, infrared radiation acts to prevent overheating of the heater and to reduce temperature gradients in the oxygen by supplying heat to the container wall.

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An Improved Method for Accelerometer Precision Centrifuge Test Data Reduction

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Introduction

THE determination of accelerometer nonlinearity coefficients from precision centrifuge tests is a difficult exercise in both test design and data reduction method. The problem is compounded when the unit under test exhibits a substantial odd second-order coefficient (i.e., a term proportional to the algebraic acceleration times the absolute value of acceleration). Twice recently TRW has tested units having this characteristic, one of them a major missile guidance accel-

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erometer. The determination of the nonlinear coefficients is particularly vital in this latter case because these terms affect targeting accuracy.

When a normal truncated power series is used to model a system, the odd quadratic nonlinearity is lumped together with the cubic term and a thoroughly erroneous error model results, particularly if the odd quadratic is due to some source different from the cubic, such as temperature. Previous methods of test and analysis used either special methods of test which provided results of questionable validity or relatively complex methods of data reduction. This Note describes a new and simple system of data reduction, named the data inversion method, that extracts the odd second-order coefficient in addition to the normal second and third-order coefficients. The method also has an inherent elimination of the zero discontinuity problem that has complicated or compromised previous methods of data reduction. It is simple enough to be programed on an on-line time-sharing computer, greatly enhancing the turnaround time for results.

Error Models

The data from any particular accelerometer test on a precision centrifuge must be fitted to some error model. A common error model used is the truncated power series:

$$A_0 = K_0 + K_1A_i + K_2A_i^2 + K_3A_i^3 \ldots \quad (1)$$

where A_0 = indicated output acceleration, A_i = input acceleration, K_0 = test and/or accelerometer bias term, K_1 = accelerometer scalefactor, and K_2,K_3 = nonlinear coefficients.

This series will fit any set of data points if enough terms are employed. The indiscriminate use of large numbers of terms, however, causes unreal higher order coefficients to be found, and worse, the nonlinear coefficients of interest will be falsified by bucking with or exchanging their values with a higher order false coefficient. For example, if one were to add a fourth-degree term to Eq. (1) and then fit it to data from a unit which had only a second-degree nonlinearity plus test noise, the data distortion due to the random noise would cause some fourth-order coefficient to be extracted to the detriment of the true second-order coefficient. This effect would become worse as the data points were fewer.

Considering the above, it is clear that one should use error models containing only terms actually present in significant amounts in the unit under test. (The condition of too few terms produces systematic residuals. The detection of too many terms is more difficult—it involves theoretical knowledge of the unit, experience with previous test results, and good judgment in looking at the data reduction results.) Historically, accelerometers were first fitted with the third-order model represented by Eq. (1). Effective separation of the second- and third-order nonlinear coefficients required data from both positive and negative input runs on the centrifuge; these terms could then be separated by their sign characteristics rather than by the small differences in shape between quadratic and cubic curves using only positive input data.

Some nonlinearities, however, produce a term proportional to acceleration times the absolute value of acceleration. Temperature effects, heating with input acceleration level, typically take this form. This term has the quadratic shape but has the sign of a cubic curve. It is therefore not possible to separate it from a third-order coefficient by its sign; the exact shape difference must be used and the results are often badly confused by test data noise. There is no complete cure for this problem. Different data reduction methods simply use different approaches using programs of varying complexity to do the best they can in extracting the coefficients from the effects of noise.

Previous Fitting Methods

Four previous approaches to coefficient determination will now be discussed. The first[1] begins by fitting each half of the data separately (positive or negative half) with a quadratic error model using the method of least squares:

$$A_0 = K_0 + K_1A_i + K_2A_i^2 \quad (2)$$

The K_0 and K_1 terms of each unipolar fit are then subtracted from the original data; what is left is the estimated curvature effect for each half. These halves are combined into one set of data points (having both positive and negative branches now) and a full third-order fit, Eq. (1), is made. The new K_2 and K_3 coefficients are the desired nonlinear terms, separated in this bipolar fit by their sign characteristics, and the K_0 and K_1 terms are small, meaningless, and discarded.

This method works very well where the true error model is represented by Eq. (1). Two difficulties arise when the final bipolar fit is to contain an odd second-order coefficient:

$$A_0 = K_0 + K_1A_i + K_2A_i^2 + K_2'A_i|A_i| + K_3A_i^3 \quad (3)$$

The first difficulty is simply one of program complication to accommodate the odd term. The second difficulty is a more subtle one and is inherent in the data reduction method. It may be called the zero discontinuity problem.

Zero discontinuity occurs when the two unipolar halves are put together for the final bipolar fit. Although the average curvatures of the two halves represented by the corresponding two unipolar K_2 terms will fair together perfectly at zero (since all K_0 and K_1 terms have been subtracted from the data), the trend of the data points themselves has no such natural restriction. Zero discontinuities will occur whenever the unipolar fits have not been made with the full error model appropriate to the accelerometer. This lack will cause the unipolar fit residuals to be systematic rather than random, but the use of the full model on unipolar halves is generally not practical due to the bucking effect between the second- and third-order terms. As shown in Ref. 1, zero discontinuities have little effect on the final result as long as the error model is represented by Eq. (1). Such discontinuities do significantly effect the results, however, when an odd second-order term is part of the model, Eq. (3). In the final fit, this odd second term will buck with the normal cubic term since they have the same signs; the zero discontinuity acts as a large extra systematic piece of noise and thereby creates a severe exchange of values even in the absence of other noise.

A manual second approach to eliminating the zero discontinuity has been tried. Graphs of the final fits are inspected and new values of K_0 and K_1 are estimated for each unipolar half to reduce the gap. The data is then reprocessed using these values for the linear terms and recalculating only the higher order terms and the results again inspected. This process is repeated until the discontinuity is eliminated.

The main drawback of this form of data reduction is that the elimination of the zero discontinuity is entirely dependent on the analyst's judgment. The final result depends to some extent on the person doing the analysis, and the many submittals to the computer cause long delays in getting the results.

Another effective method of data reduction[2] fits all the data at once by allowing independent linear coefficients for the positive and negative halves:

$$A_0 = K_0^+ + K_0^- + K_1^+A_i^+ + K_1^-A_i^- + K_2A_i^2 + K_2'A_i|A_i| + K_3A_i^3 \quad (4)$$

Since it fits only once with the full error model, most zero discontinuity effects are avoided. The K_2', K_3 bucking effect is present but that is inherent in the test, not the data reduction method. Any systematic error term not included in the equation will cause additional bucking trouble, but that is generally true of any system. The main objection to this method is the complex computer program required.

A fourth approach is to try to design the test program itself to avoid an odd second-order term. If the term is due to some temperature effect, for example, it might be possible to build special temperature stabilizing fixturing, perform the measurements before significant changes have taken place, or perform the test and measurements in such a way so as to be able to extrapolate the data back to some standard temperature point. All of these methods have been tried with temperature-sensitive units, almost always with poor results because of the extreme difficulty of controlling the parameters to the degree needed.

Data Inversion Method

The data inversion method was developed with the dual purpose of reducing the zero discontinuity problem when extracting an odd second-order coefficient, and creating a program for the time-sharing system at TRW to provide quick turnaround of data reduction.[3] In essence, this method follows the earlier concept of unipolar half fitting.[1] It then determines the normal second-order coefficient K_2 directly from the unipolar results, removes this term from the data, inverts the negative half of the data by changing input and output signs, and then fits the combined data (all positive now) with Eq. (1). A detailed description of the steps and rationale follows.

The input data is first processed to put it in the form of input and output accelerations. Centrifuge periods are combined with the nominal centrifuge arm radius, the measured arm stretch, and the local gravity to compute the input acceleration. The accelerometer output is combined with the unit scalefactor and the output acceleration for each point is computed. The output accelerations are then reduced by the subtraction of the calculated input accelerations for each point. These points, inputs vs residual outputs, are then fitted with Eq. (2). This least-squares curve fitting is performed on the negative input data and the positive data separately, and in each case the three coefficients K_0, K_1, K_2 are determined. Since the K_0 and K_1 coefficients represent errors in centrifuge arm radius, misalignments, and off-nominal scalefactors, and since they do not affect the nonlinear terms, they may be discarded. At this point, the final second-order nonlinear coefficient can for all practical purposes be determined directly from the two second-order terms of the unipolar fits; it is the algebraic average of these coefficients. (This assumes that there is no even third-order coefficient combining with the quadratic term—the author has never seen evidence of such a term with any significant size.) This method of determining the final second-order coefficient has been checked by comparison with many previous runs using the normal TRW program[1] and it results in differences from the original determinations of less than 0.1 $\mu g/g^2$. After calculating this coefficient, the program computes the effect of this term on all the points of the unipolar fits and extracts this effect from the output data. This operation results in the positive and negative halves of the data being balanced, i.e., having the same total amount of curvature. The signs of both input and output of the negative unipolar data are then reversed; the negative data is inverted. This operation joins the positive and negative data into one consistent series of positive acceleration points. This series of points is then fitted with Eq. (1). The quadratic coefficient from this fit is the desired odd second-order coefficient and the cubic coefficient is, of course, the final third-order coefficient.

The method of inverting the negative unipolar half and then making a unipolar fit to exclusively positive data eliminates most normal discontinuities between the positive and negative halves at zero input acceleration. Such discontinuities almost always come about either from small unmodeled error sources or from the use of a restricted model for unipolar fitting, and the resulting systematic discontinuity automatically disappears with the data inversion.

Comparison of Results

Three programs were used to reduce the same data from two accelerometers. The three programs compared were those that tend to minimize the zero discontinuity problem, the data inversion method, the manual iterative approach, and the Autonetics program. K_2 coefficients agreed within 0.2 $\mu g/g^2$, K_2' coefficients within 1.5 $\mu g/g^2$, and K_3 coefficients within 0.03 $\mu g/g^3$.

This correspondence between the nonlinear terms is easily within the uncertainty of the data. It must be emphasized that a meaningful separation of an odd second-order coefficient from a normal third is difficult at best. Large numbers of data points are desirable as are repeat runs and averages between instruments.

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